Tests of the Oktay-Kronfeld Action with b-meson Spectrum

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Motivation: ε_K

- In flavor physics, we are interested in the CKM matrix element V_{cb}.
- V_{cb} enters to the theoretical (SM) expression for ε_K which quantifies the indirect CP violation.
- 3.4σ tension can be observed using most up to date input parameters from lattice: $\hat{B}_K$, exclusive V_{cb}. *Preliminary*

$$|\varepsilon_K|^{\text{exp}} = 2.228(11) \times 10^{-3} \qquad \text{(PDG)}$$

$$|\varepsilon_K|^{\text{SM}} = 1.570(195) \times 10^{-3} \qquad (\text{SWME } \hat{B}_K, \text{FNAL/MILC } V_{cb})$$

- This tension goes away, if we use inclusive V_{cb}.

Motivation: heavy quarks on the lattice

- The dominant error in theoretical determination of ϵ_K comes from V_{cb}.

$$\begin{cases} 33.7\% & \leftarrow V_{cb} \\ 19.7\% & \leftarrow \hat{B}_K \end{cases}$$

$\Rightarrow$ More precise V_{cb} might lead to larger tension on ε_K.

- Combining HFAG average of experimental results and lattice form factor $\mathcal{F}$ calculation of the semi-leptonic decays, we can extract exclusive V_{cb}.

$$\bar{B} \to D^* l \nu_l \,,\ \bar{B} \to D l \nu_l$$

- Because the dominant error for the form factor $\mathcal{F}$ calculation is heavy quark discretization error, we need an highly improved lattice action or finer lattice ensemble.
- OK action was designed as an improved action.
- Here, we will verify the improvement in b-meson spectrum.

OK Action

- OK action is desigend to improve the heavy quark action of Fermilab formulation. $S_{\text{Fermilab}} = S_0 + S_B + S_E$.

$[\lambda \sim a\Lambda,\ \Lambda/m_Q]$

$$S_0 = m_0 \sum_x \bar{\psi}(x)\psi(x) + \sum_x \bar{\psi}(x)\gamma_4 D_4 \psi(x) \qquad : \mathcal{O}(1)$$

$$+ \zeta \sum_x \bar{\psi}(x)\vec{\gamma}\cdot\vec{D}\psi(x) - \tfrac{1}{2}a \sum_x \bar{\psi}(x)\triangle_4\psi(x) - \tfrac{1}{2} r_s \zeta a \sum_x \bar{\psi}(x)\triangle^{(3)}\psi(x)$$

$$S_B = -\tfrac{1}{2} c_B \zeta a \sum_x \bar{\psi}(x) i\vec{\Sigma}\cdot\vec{B}\psi(x) \qquad : \mathcal{O}(\lambda)$$

$$S_E = -\tfrac{1}{2} c_E \zeta a \sum_x \bar{\psi}(x)\vec{\alpha}\cdot\vec{E}\psi(x) \quad (c_E \neq c_B : \text{ OK action}) \qquad : \mathcal{O}(\lambda^2)$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

[A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

OK Action

- OK action includes dim-6 and -7 operators necessary for tree-level matching to QCD through order $\mathcal{O}(\lambda^3)$: $S_{\mathrm{OK}} = S_{\mathrm{Fermilab}} + S_{\mathrm{new}}$

$$
\begin{aligned}
S_{\mathrm{new}} = & \; c_1 a^2 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_i \triangle_i \psi(x) \\
& + c_2 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, \triangle^{(3)}\} \psi(x) \\
& + c_3 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, i\vec{\Sigma} \cdot \vec{B}\} \psi(x) \\
& + c_{EE} a^2 \sum_x \bar{\psi}(x) \{\gamma_4 D_4, \vec{\alpha} \cdot \vec{E}\} \psi(x) \\
& + c_4 a^3 \sum_x \bar{\psi}(x) \sum_i \triangle_i{}^2 \psi(x) \\
& + c_5 a^3 \sum_x \bar{\psi}(x) \sum_i \sum_{j \neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \psi(x) \qquad : \mathcal{O}(\lambda^3)
\end{aligned}
$$

Meson Correlator

$$C(t, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}^\dagger(t, \boldsymbol{x}) \mathcal{O}(0, \boldsymbol{0}) \rangle$$

- On the lattice, we calculate the 2-point correlator.
- valence heavy quark $\psi(x)$: tadpole improved OK action ($\tilde{\kappa} = 0.038, 0.039, 0.040, 0.041$)
- valence light quark $\chi(x)$: asqtad staggered action ($am_q = am'_s$)
- 11 meson momenta $|\boldsymbol{p}a| (= 0,\ 0.099,\ \cdots,\ 1.26)$ for dispersion fit
- MILC asqtad $N_f = 2 + 1$ ensemble

a(fm)	$N_L^3 \times N_T$	β	am'_l	am'_s	u_0	a^{-1}(GeV)	N_{conf}	$N_{t_{\text{src}}}$
0.12	$20^3 \times 64$	6.79	0.02	0.05	0.8688	1.683^{+43}_{-16}	484	6
0.15	$16^3 \times 48$	6.60	0.029	0.0484	0.8614	1.350^{+35}_{-13}	500	4

Interpolating Operator

- Meson correlator

$$C(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}^\dagger(t,\boldsymbol{x})\mathcal{O}(0,\boldsymbol{0})\rangle$$

- Heavy-light meson interpolating operator

$$\mathcal{O}_{\mathtt{t}}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\Omega_{\beta\mathtt{t}}(x)\chi(x)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{(pseudoscalar)} \\ \gamma_\mu & \text{(vector)} \end{cases}, \ \Omega(x) \equiv \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$$

- Quarkonium interpolating operator

$$\mathcal{O}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)$$

[Wingate *et al.*, PRD **67**, 054505 (2003) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

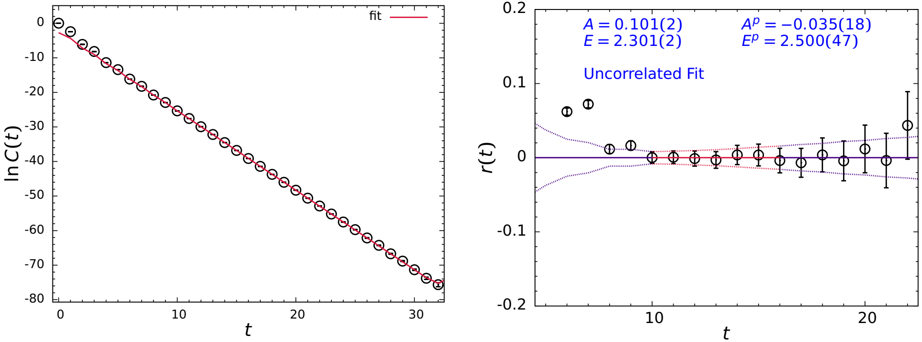
Correlator Fit: extract the energy E

- fit function ($A^p \equiv 0$ for quarkonium)

$$f(t) = A\{e^{-Et} + e^{-E(T-t)}\} + (-1)^t A^p\{e^{-E^p t} + e^{-E^p(T-t)}\}$$

- fit residual

$$r(t) = \frac{C(t) - f(t)}{|C(t)|}\text{, where } C(t) \text{ is data.}$$

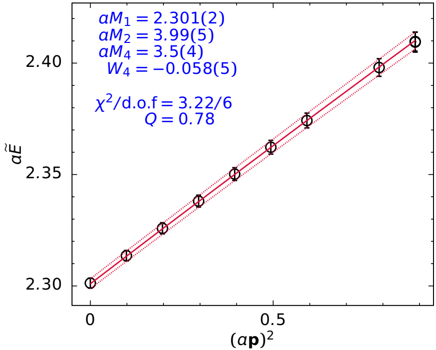


$[\overline{Q}q$, PS, $\tilde{\kappa} = 0.038$, $\boldsymbol{p} = \mathbf{0}]$

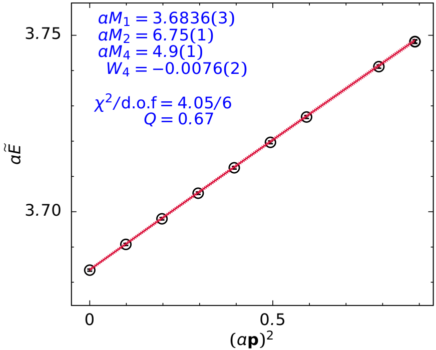
Dispersion Fit: extract the masses M_1 and M_2

$$E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} - \frac{(\boldsymbol{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_i p_i^4$$

$$\widetilde{E} = E + \frac{a^3 W_4}{6}\sum_i p_i^4 , \quad \boldsymbol{n} = (2,2,1)\,,(3,0,0)$$



$[\overline{Q}q$, PS, $\tilde{\kappa} = 0.038]$



$[\overline{Q}Q$, PS, $\tilde{\kappa} = 0.038]$

Improvement Test: Inconsistency Parameter

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$

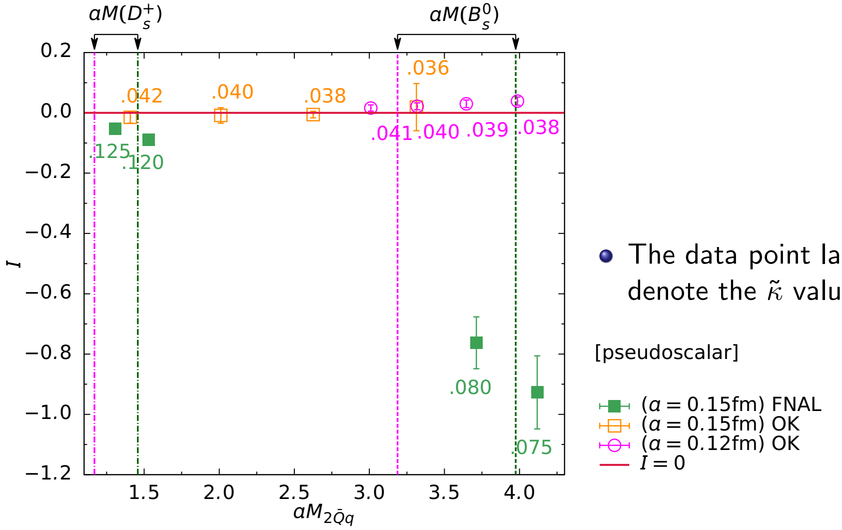
$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

[S. Collins *et al.*, NPB **47**, 455 (1996) , A. S. Kronfeld, NPB **53**, 401 (1997)]

- By design, the inconsistency parameter I can examine the action improvements by $\mathcal{O}(\boldsymbol{p}^4)$ terms. I isolate the δB of $\mathcal{O}(\boldsymbol{p}^2)$ effect.
- In the continuum limit, $B_1 = B_2$ and I vanishes.
- By including up to $\mathcal{O}(\boldsymbol{p}^4)$, the OK action is closer to the renormalized trajectory S_{RT} than the Fermilab action.
- We expect I is close to 0.

Improvement Test: Inconsistency Parameter

- Near B_s^0 mass, the coarse ($a = 0.12$fm) ensemble data shows significant improvement compared to the Fermilab action.



- The data point labels denote the $\tilde{\kappa}$ values.

Improvement Test: Hyperfine Splitting Δ

- The difference in hyperfine splittings $\Delta_2 - \Delta_1$ also can be used to examine the improvement from $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

$$\Delta_1 = M_1^* - M_1 \,,\ \Delta_2 = M_2^* - M_2$$

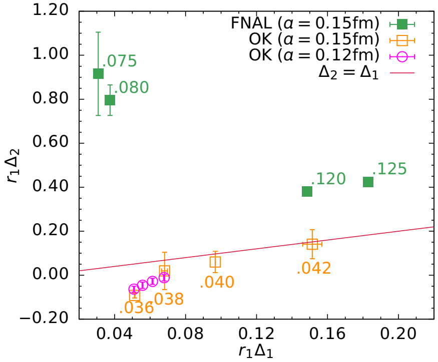
$$\begin{aligned} M^{(*)}_{1\overline{Q}q} &= m_{1\overline{Q}} + m_{1q} + B^{(*)}_{1\overline{Q}q} \\ M^{(*)}_{2\overline{Q}q} &= m_{2\overline{Q}} + m_{2q} + B^{(*)}_{2\overline{Q}q} \\ \delta B^{(*)} &= B_2^{(*)} - B_1^{(*)} \end{aligned}$$

$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$

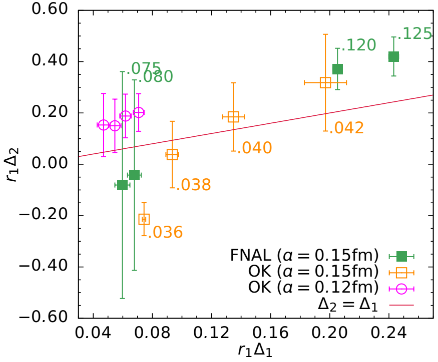
- In the continuum limit $\Delta_2 = \Delta_1$.

Improvement Test: Hyperfine Splitting Δ

- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- The heavy-light results do not deviate much from the line $\Delta_2 = \Delta_1$ even with the Fermilab action, and remain in good shape with the OK action.



Quarkonium



Heavy-light

Conclusion and Outlook

- Inconsistency parameter shows that the OK action clearly improves $\mathcal{O}(\boldsymbol{p}^4)$ terms.
- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- For heavy-light system, both OK and Fermilab action does not deviate much from the line $\Delta_2 = \Delta_1$. But we can see the statistical gain from the results of OK action.
- We plan to calculate form factor of the decay $\bar{B} \to D^* l\nu$ by using OK action. Then, we can determine exclusive V_{cb} with higher precision.
- Lattice current improvement to the same order of the OK action is under way.
- We plan on the 1-loop improvement for the coefficients c_B and c_E in the OK action.
- We plan on the development of highly optimized CG inverter using QUDA (GPU computing).

Thank you for your attention.

Effective Mass

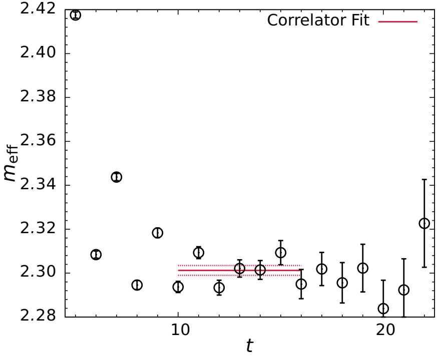
$$m_{\text{eff}}(t) = \frac{1}{2} \ln\left(\frac{C(t)}{C(t+2)}\right)$$

For small t,

$$C(t) \cong A(e^{-Et} + \beta e^{-(E+\Delta E)t})$$
$$= Ae^{-Et}(1 + \beta e^{-(\Delta E)t}) \,,$$

$$\begin{cases} \beta > 0 & \text{(excited state)} \\ \beta \sim -(-1)^t & \text{(time parity state)} \end{cases}$$

$$m_{\text{eff}} \approx E + \beta(\Delta E)e^{-(\Delta E)t}$$



$[\overline{Q}q,\ \text{PS},\ \tilde{\kappa} = 0.038,\ \boldsymbol{p} = \boldsymbol{0}]$

Improvement Test: Inconsistency Parameter

- Considering non-relativistic limit of quark and anti-quark system, for S-wave case ($\mu_2^{-1} = m_{2\overline{Q}}^{-1} + m_{2q}^{-1}$),

$$\begin{aligned} \delta B_{\overline{Q}q} &= \frac{5}{3}\frac{\langle \boldsymbol{p}^2 \rangle}{2\mu_2}\left[\mu_2\left(\frac{m_{2\overline{Q}}^2}{m_{4\overline{Q}}^3} + \frac{m_{2q}^2}{m_{4q}^3}\right) - 1\right] \qquad (m_4 : c_1, c_3) \\ &+ \frac{4}{3}a^3\frac{\langle \boldsymbol{p}^2 \rangle}{2\mu_2}\mu_2(w_{4\overline{Q}} m_{2\overline{Q}}^2 + w_{4q} m_{2q}^2) \quad (w_4 : c_2, c_4) \\ &+ \mathcal{O}(p^4) \end{aligned}$$

[A. S. Kronfeld, NPB **53**, 401 (1997) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

- Leading contribution of $\mathcal{O}(\boldsymbol{p}^2)$ in δB vanishes when $w_4 = 0$, $m_2 = m_4$, not only for S-wave states but also for higher harmonics.